APPENDIX 1

Derivation of matrix equation (1)

In the October 1 census each semipalmated sandpiper is tallied either as a juvenile or as an adult. Adults that over-summered (denoted A_o) or those that made a breeding migration to and from the Arctic (denoted A_m) cannot be distinguished during the census, and are tracked separately in the model because their survival and reproduction differ strongly. Yearlings are not represented directly in the census, as they by definition graduate to adulthood just prior to the census.

Juveniles are born in the Arctic, and make a southward migration to non-breeding areas at ~4 months of age, arriving before the census is made. The juvenile classification extends through the following non-breeding season until April 1 (i.e. until they are ~10 months of age) when they by definition become *yearlings*. They decide on this date either to make a breeding migration to and from the Arctic, or to over-summer on the non-breeding area. During the summer (April 1 – September 30) the model tracks four classes, namely over-summering and migrating yearlings (Y_o, Y_m), and over-summering and migrating adults (A_o, A_m). Yearlings by definition become *adults* at the end of summer, and in the October 1 census are assigned to A_o if they over-summered, and to A_m if they migrated.

The census in year *t* is denoted by the vector

$$\begin{pmatrix} J(t) \\ A_0(t) \\ A_m(t) \end{pmatrix}$$

The matrix equation is

$$\begin{pmatrix} J(t+1) \\ A_0(t+1) \\ A_m(t+1) \end{pmatrix} = M \begin{pmatrix} J(t) \\ A_0(t) \\ A_m(t) \end{pmatrix} \qquad (1)$$

The derivation of the matrix *M* is as follows. It is the product of four smaller matrices:

$$M = \begin{bmatrix} breeding\ plus \\ southward\ survival\ to \\ wintering\ grounds \end{bmatrix} \begin{bmatrix} survival \\ during\ northward \\ migration \end{bmatrix} \begin{bmatrix} decision\ to \\ migrate \\ on\ April\ 1 \end{bmatrix} \begin{bmatrix} survival \\ from\ October\ 1 \\ to\ March\ 31 \end{bmatrix}$$

or

or

$$M = M_4 M_3 M_2 M_1$$

The matrix M_1 models the winter (October – March) survival of the three census classes (juveniles J, adults that over-summered A_o, adults that migrated A_m):

$$M_1 = \begin{pmatrix} S_{JN} & 0 & 0 \\ 0 & S_{AN} & 0 \\ 0 & 0 & S_{AN} \end{pmatrix}$$

At the end of winter (April 1) juveniles by definition become yearlings, and along with surviving adults are assigned to over-summering and migrating classes with the measured probabilities. Adults over-summer with probability p_{Ao} and migrate with probability $1 - p_{Ao}$. Yearlings over-summer with probability p_{Yo}, and migrate with probability $1 - p_{Yo}$. During the summer the model tracks these four classes of individuals in matrix M_2:

$$M_2 = \begin{array}{c} decision\ on\ April\ 1st \\ \begin{pmatrix} p_{Yo} & 0 & 0 \\ 1 - p_{Yo} & 0 & 0 \\ 0 & p_{Ao} & p_{Ao} \\ 0 & 1 - p_{Ao} & 1 - p_{Ao} \end{pmatrix} \end{array}$$

At the start of summer (indexed as (t + 0.5)) the numbers of these classes can be calculated as the product of M_1 and M_2 and the fall census

$$\begin{pmatrix} Y_o(t+0.5) \\ Y_m(t+0.5) \\ A_0(t+0.5) \\ A_m(t+0.5) \end{pmatrix} = M_2 M_1 \begin{pmatrix} J(t) \\ A_0(t) \\ A_m(t) \end{pmatrix}$$

Successful breeding requires that yearling migrants survive the northward migration to the Arctic. This is estimated as the square root of the survival of the return migration (S_{Ym} which was measured). The survival of over-summering yearlings and adults during this half of the summer is estimated as the square root of their (measured) over-summering survival S_{Yo} and S_{A0}. This step is denoted in matrix M_3:

$$M_3 = \begin{matrix} \\ \\ \\ \\ \end{matrix} \overset{survival\ during\ migration}{\begin{pmatrix} \sqrt{S_{Y0}} & 0 & 0 & 0 \\ 0 & \sqrt{S_{Ym}} & 0 & 0 \\ 0 & 0 & \sqrt{S_{A0}} & 0 \\ 0 & 0 & 0 & \sqrt{S_{Am}} \end{pmatrix}}$$

In the final step, the reproduction of migrants and the survival of both migrants and over-summering birds is accounted. The expected reproduction of migrant yearlings is $F_Y\sqrt{S_{Ym}}$, and the expected reproduction of adult migrants is $F_A\sqrt{S_{Am}}$. All birds must survive the second half of the summer, denoted in matrix M_4:

$$M_4 = \begin{matrix} \\ \\ \\ \end{matrix} \overset{breeding\ and\ survival\ till\ October}{\begin{pmatrix} 0 & S_J F_Y & 0 & S_J F_A \\ \sqrt{S_{Y0}} & 0 & \sqrt{S_{A0}} & 0 \\ 0 & \sqrt{S_{Ym}} & 0 & \sqrt{S_{Am}} \end{pmatrix}}$$

The full matrix M is the product $M_4 M_3 M_2 M_1$ in which all periods are combined:

$$M = \begin{pmatrix} S_J F_Y \sqrt{S_{Ym}}(1-p_{Yo})S_{JN} & S_J F_A \sqrt{S_{Am}}(1-p_{Ao})S_{AN} & S_J F_A \sqrt{S_{Am}}(1-p_{Ao})S_{AN} \\ S_{Yo} p_{Yo} S_{JN} & S_{Ao} p_{Ao} S_{AN} & S_{Ao} p_{Ao} S_{AN} \\ S_{Ym}(1-p_{Yo})S_{JN} & S_{Am}(1-p_{Ao})S_{AN} & S_{Am}(1-p_{Ao})S_{AN} \end{pmatrix}$$